Romanian Journal of Regional Science
Vol. 18, No. 1, Summer 2024

DOI: https://doi.org/10.61225/rjrs.2024.02

NATIONAL WELFARE IMPLICATIONS OF REGIONAL CHILDCARE POLICY: A THEORETICAL APPROACH

Hiroyuki Hashimoto [a,*] , Tohru Naito [b,c]

[a] University of Hyogo, Japan

[b] Doshisha University, Japan

[c] George Mason University, USA

* Corresponding author

Address: 8-2-1 Gakuennishi-machi Nishi-ku, 6512197 Japan

E-mail: hiro@mba.u-hyogo.ac.jp

Biographical Notes

Hiroyuki HASHIMOTO is an associate professor in the Graduate School of Social Sciences, Department of Professional Business, University Hyogo. His research interests are population economics, regional economics, and economic growth theory. ORCID ID: https://orcid.org/0000-0002-9458-8583

Tohru NAITO is a professor in Graduate School of Commerce in Doshisha University. His research interest are spatial economics, environmental economics and urban economics. He is also interested in privatization of public firm. ORCHID ID: https://orcid.org/0000-0002-5996-198

Abstract

This study examines the effects of regional childcare policy on regional and total fertility, interregional migration, capital accumulation, and welfare. The study utilizes an overlapping generations model with endogenous fertility and two asymmetric regions; one region has children who do not have access to childcare facilities, whereas the other region does not. In this setting, Hashimoto and Naito (2023) showed that a regional childcare policy can increase both regional and total fertility. However, they did not refer to welfare effects. As the government's ultimate objective is to maximize or improve social welfare, it need not necessarily focus on overcoming declining fertility rates. Therefore, this study explores whether the childcare support policy of Hashimoto and Naito (2023) has improved the social welfare of the economy. Under a plausible rate of labor income share, the childcare support policy raises consumption owing to a decrease in capital accumulation. Thus, childcare support policy increases social welfare when the fertility rate rises or is unaffected, and it does not necessarily increase social welfare if it causes a decline in the fertility rate.

Keywords: childcare policy, welfare, fertility, overlapping generations model, capital accumulation
JEL Classification: E61, J13, J18, R23

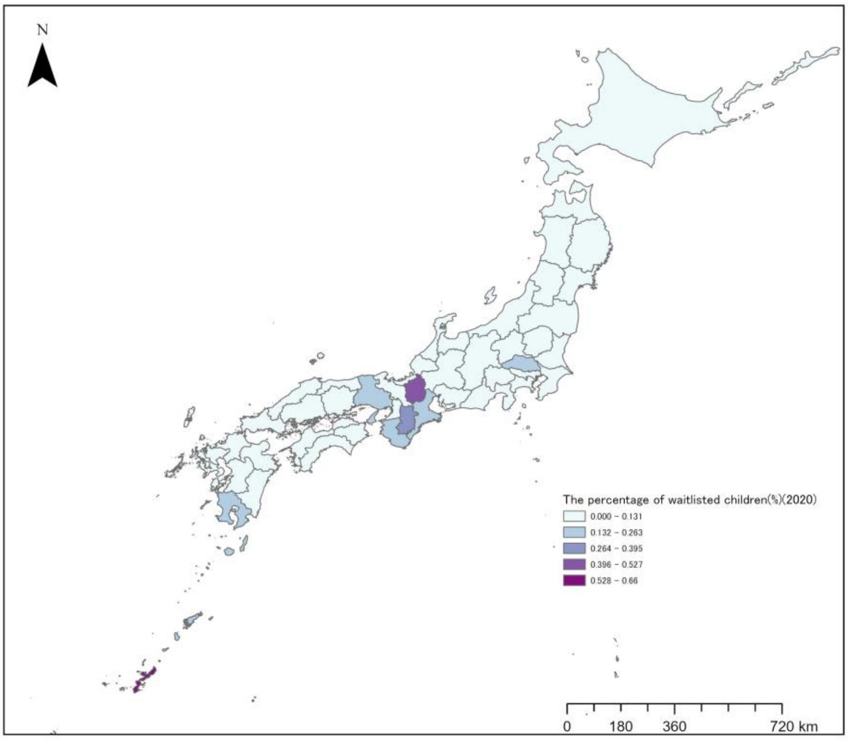
1. Introduction

Low fertility is a prevalent phenomenon in most developed countries. Low fertility rates are expected to lead to a population decline, as observed in countries such as Japan. Population decline has various negative impacts on the economy: smaller market size, decreased labor force, deterred economic growth, and consequently, decreased consumption and savings. Whether these effects exist is highly empirical, but a decrease in the population of a society indicates that the society will eventually disappear, and ceteris paribus, tax revenues will decrease. Therefore, population decline is an economic problem that governments at various levels need to overcome. The female labor force is expected to compensate for the resulting labor shortages; hence, various private and public support measures have been implemented to increase women's labor participation and employment rates.

According to Statistics Bureau of Ministry of Internal Affairs and Communications in 2024, dual-income families accounted for 71% of all families in 2023, up from 38% in 1980. It is interesting to determine who raises children in countries with declining fertility rates. In general, childcare obligations seem to impose a greater burden on women than on men in these countries. *Waitlisted children* are those who cannot access to adequate childcare facilities.[1] Households that are unable to leave their children in adequate childcare facilities tend to face a decrease in available working hours. When families are unable to find adequate childcare facilities for their children, it gives rise to the problem of waitlisted children. Figure 1 shows the distribution of waitlisted children among prefectures in Japan in 2020. Overall, 0.45% of children in 2020 were waitlisted in prefectures comprising large cities, which are labelled as urban areas, compared with 0.41% for prefectures without large cities. The ratio of waitlisted children is higher in urban areas than in rural ones.

[1] For the term "Waitlisted children," please see the following website: https://www.mhlw.go.jp/english/org/pamphlet/dl/serviceguide2022.pdf

Figure 1. Percentage of waitlisted children among prefectures in Japan, 2020

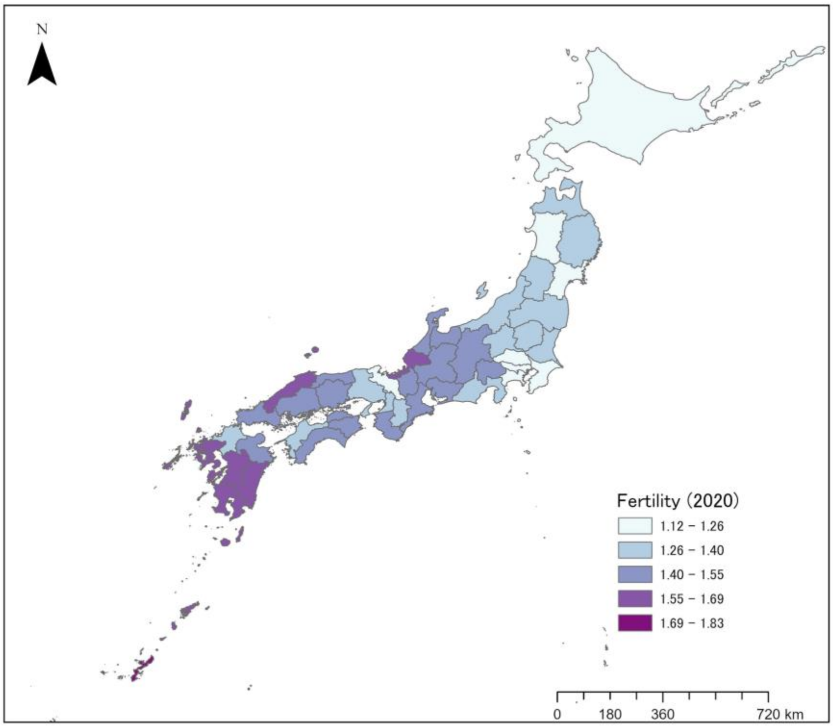


Source: Children and Families Agency in Japan **(**https://www.cfa.go.jp/policies/hoiku/torimatome/r5)

Figure 2 indicates the fertility rates for each prefecture in Japan in 2020. There are differences in both the fertility rates and percentage of waitlisted children among prefectures. Furthermore, lower fertility rates have been observed in urban areas. For instance, prefectures such as Tokyo, Osaka, and Kanagawa, which include major Japanese metropolises, exhibit lower fertility rates, whereas the fertility rates in prefectures such as Miyazaki, Shimane, and Fukushima, which are geographically distant from Japan's metropolitan areas, are relatively high. Figures 1 and 2 jointly suggest a negative relationship between the ratio of waitlisted children and fertility rate in Japan, although other factors are not controlled for.

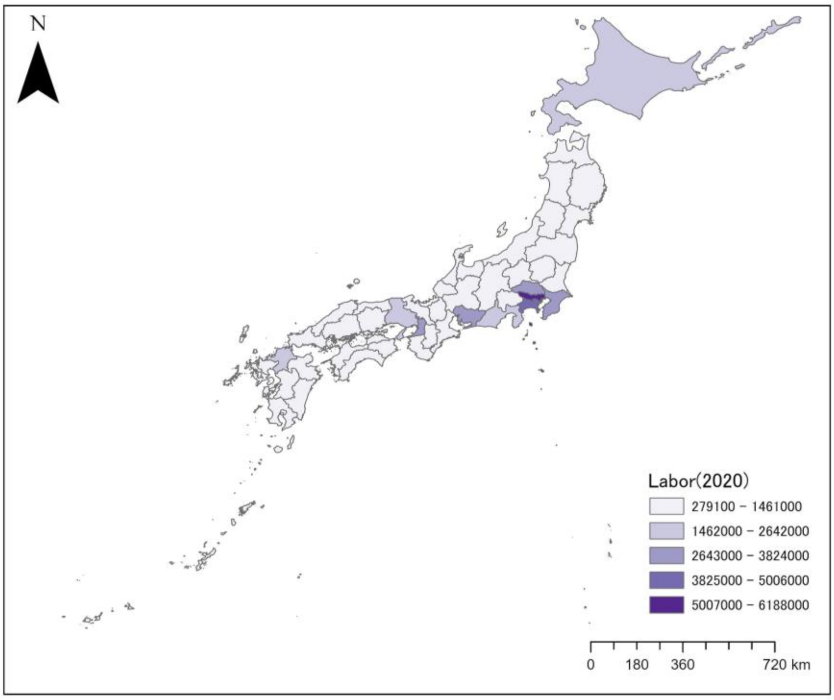
The distribution of the working population across Japan also exhibits a significant variation. Figure 3 indicates the percentage of the working-age population in each prefecture to the total working-age population of Japan in 2020. However, this percentage is relatively high in urbanized areas such as the Tokyo and Kansai metropolitan areas, which have Osaka and Kyoto as the core city, respectively.

Figure 2. Fertility rates for each prefecture of Japan, 2020



Source: Ministry of Health, Labour and Welfare in Japan (https://www.mhlw.go.jp/index.html)

Figure 3. Labor force of prefectures in Japan, 2020



Source: Statistics Bureau of Ministry of Internal Affairs and Communications in Japan **(**https://dashboard.e-stat.go.jp/)

A large working-age population in an urban region reduces wages in that region and simultaneously increases them through concentration effects. In addition, congestion generally occurs in residential and work regions where the population is concentrated. The issue of waitlisted children is a typical example of this congestion phenomenon. It is necessary to account for differences in the distribution of the working-age population when the interregional gap in fertility rate or waitlisted children are investigated because family planning decisions are made by adults who are generally also of working age.

Extensive research has been done on the relationship among fertility, population, and the regional economy. Using a two-region overlapping generations model, Sato and Yamamoto (2005) showed that a decline in child mortality results in a decrease in the fertility rate, leading to a demographic transition. Using a multi-region overlapping generations model, Sato (2007) demonstrated that a concentrated economy causes an increase in population, leading to congestion, which explains the decrease in fertility. Goto and Minamimura (2019), by using a multi-country overlapping generations model, found that economic integration can lead to population concentration in larger countries, a decrease in fertility in these countries, and a decrease in population size in the long run. Yakita(2011) introduced public goods into an overlapping generations model with two regions and an asymmetric production technology, and analyzed how the benefits from spillovers of regional public goods affect demographics. Several studies have empirically analyzed regional concentration or fertility. Koka and Rapallini (2023) examined how the aging of the population in Italy affects policies regarding childcare assistance. They used a stochastic voting model with commitments to estimate the desired policy and showed childcare subsidies positively affect fertility rates and improve welfare. Schoppa (2020) examined how Japan has adopted this menu of policies over the past 30 years in the hopes of increasing its fertility rate. Although previous studies have explored interregional migration and regional fertility, focusing on various features of the regional economy that exist, the issue of waitlisted children has not yet been extensively studied. Some exceptions are as follows. Kawabata(2014) empirically clarified that accessibility to childcare facilities makes it difficult for women with children to get the jobs they want.

Hashimoto and Naito (2021) used a two-region overlapping generations model with endogenous fertility to study the effect of a decrease in the exogenous probability of a child becoming a waitlisted child. Hashimoto and Naito (2024) analyzed the childcare policy that decreases the probability of a child becoming a waitlisted child, which is determined endogenously. Although they analyzed the impact of childcare policy on regional fertility, total fertility, interregional migration, and capital accumulation, they did not mention social welfare. Therefore, this study conducts a welfare analysis.

The remainder of this paper is organized as follows. Section 2 presents a two-region overlapping generations model of endogenous fertility that incorporates waitlisted children, interregional migration, and government behavior. Section 3 analyzes capital accumulation and population distribution in equilibrium. Section 4 examines the effects of childcare policy on regional fertility, interregional migration, and capital accumulation. Section 5 presents the welfare analysis and discussion. Section 6 concludes the paper.

2. The model

To incorporate the regional differences in fertility and labor force mentioned in the previous section, we develop the multi-region model to analyze the impact of regional childcare policy on social welfare theoretically. We consider a closed economy with two regions, which is populated by overlapping generations of individuals who live for three periods. Each individual chooses to live in either region u or region r and makes a fertility choice.

2.1 Individuals

In the first period, the individuals are children who make no decisions.[2] They become adults in the second period and engage in decision-making; they choose whether to reside in region u or r at the beginning of this period, whether to have children, how to allocate their time between work and childrearing, and whether to save the income they earn. The migration caused by changing the region of residence is assumed to occur only once in life; once individuals reside in a region during the second period, they remain there in the third period. In the third period, they retire and consume all of their savings. Adults derive utility from the number of their children and their own consumption during retirement. Thus, the adult population in any period t consists of the adult population in regions u and r, $N_t = N_t^u + N_t^r$, where N_t and N_t^i represent the adult population in period t and the adult population in the region $i(= u, r)$, respectively. The preferences of an adult of region i in period t are defined based on the number of children n_t^i and consumption c_{t+1}^i in the retirement period. Assuming that all individuals have a common preference, the utility function is given by

$$U_t^i = \gamma \ln n_t^i + (1-\gamma) \ln c_{t+1}^i, \quad 0 < \gamma < 1, \ (i = u, r) \tag{1}$$

[2] In spite that individuals in the childhood don't make any decisions, we explicitly mention the existence of the childhood period, because it is useful for intuitive understanding of the structure of the model and results being derived. It is notable that the structure of the mathematical optimization problem is no different from the standard two-period OLG model, where the lifetime period is explicitly stated as two periods.

where γ reflects fertility preferences and $1-\gamma$ indicates consumption preferences. We assume that parenting is time-consuming. Each individual allocates their time between childrearing and work; an increase in childrearing time means a decrease in work time, which in turn leads to a decrease in labor income at any given wage rate. We also assume that childrearing time is proportional to the number of children, specified as $z_t^i n_t^i$, where $z_t^i \in (0,1)$ denotes the unit childcare cost in period t in region i. z_t^i is generally supposed to depend on the availability of public childcare services and other personal and social circumstances, including whether one works from home, transportation status, and has familial support. The childrearing environment varies from region to region; therefore, z_t^u and z_t^r generally differ. As Hashimoto and Naito(2023) stated, the supply of childcare services is insufficient to satisfy the demand in Japan; this problem is particularly acute in urban areas. When the supply of childcare services is insufficient, a disparity arises, with some parents being able to access them and others being unable to do so. [3] We assume that the supply of childcare facilities is insufficient in region u and that, consequently, access to the facilities is assigned to a portion of the adults there. The unit childcare cost in region u is denoted by z if the parents do not have access to their desired childcare facilities and by μz if they can access the childcare facility they prefer, where $\mu \in (0,1)$. If the assigned childcare facility does not support the parents' work schedules, they will be forced to significantly reduce their working time when they have children. [4] μ stands for the inefficacy of childcare facilities in supporting working parents; a larger μ implies that the parent incurs greater childcare costs. [5] We assume that the assignment of childcare facilities is probabilistic and that parents face a situation in which the facilities are not available with standby probability $p_t \in (0,1)$. The more childcare facilities there are, the more likely it is that childcare facilities will be available. Thus, the standby probability p_t can be considered a function of the sufficiency of childcare facilities in region u, denoted by f_t; $p_t = p(f_t)$. f_t increases as the government in region u, ceteris paribus, increases the supply of public childcare facilities. The parents in region u face a unit childcare cost of z with probability $p(f_t)$ and μz with probability $1-p(f_t)$ when determining the number of children they will have at the beginning of period t. Thus, the unit childcare cost in region u, z_t^u, is defined as a linear combination of the two situations, weighted by each probability:

$$z_t^u = [p(f_t) + (1-p(f_t))\mu]z. \qquad (2)$$

[3] For an account of regional differences in the waitlisted children issue, see Hashimoto and Naito(2021)

[4] Childcare facilities are less useful for parents when they open late, close early, or frequently call the parents in for various reasons.

[5] In general, parents may be classified in one of two ways: those who do not want to utilize childcare facilities and those who want to work part-time or full-time and would like to utilize childcare facilities. The study model considers only the latter.

In contrast to region u, we assume that, in region r, childcare facilities meet the demand such that all parents can use them if they desire. For the model to be tractable, the childcare cost in region r, z_t^r, is assumed to be common across parents and constant over time, $\bar{z}$. [6] The budget constraints for each adult in region u in period t are given by

$$(1-[p(f_t)+(1-p(f_t))\mu]zn_t^u)w_t^u = s_t^u \tag{3}$$

and

$$R_{t+1}s_t^u = c_{t+1}^u, \tag{4}$$

where n_t^u, w_t^u, and s_t^u are the number of children, wage, and savings in region u, respectively. R_{t+1} and c_{t+1}^u denote the gross rates of return on capital and consumption during retirement, respectively. The rate of return on capital is the same for the two regions because we assume that the capital market is integrated within the economy. By maximizing (1) subject to (3) and (4) with respect to n_t^u and c_{t+1}^u, we obtain

$$n_t^u = \frac{\gamma}{[\mu+(1-\mu)p(f_t)]z} \tag{5}$$

and

$$c_{t+1}^u = R_{t+1}(1-\gamma)w_t^u. \tag{6}$$

(5) shows that a higher standby probability $p(f_t)$ results in fewer children.[7] The budget constraints of the individuals in region r are given by

$$(1-\bar{z}n_t^r)w_t^r = s_t^r \tag{7}$$

and

$$R_{t+1}s_t^r = c_{t+1}^r. \tag{8}$$

---

[6] We also posit the existence of insurance companies that behave competitively. With the assumption that the administrative costs per contract are negligible, payments are made on an actuarially fair basis. Each individual in region u buys actuarially fair insurance before they make decisions.

[7] We set up the model in such a way that the determination of fertility is not affected by the wage rate. Then, the role of the accessibility to childcare service in the determination of fertility can be investigated with mathematical clarity, and the set makes it easier to derive clear policy implication conducting.

To simplify the analysis, we assume $\bar{z}$ is equal to μz. By maximizing (1) subject to (7) and (8) with respect to n_t^r and c_{t+1}^r, we obtain the optimum as follows.

$$n_t^r = \frac{\gamma}{\bar{z}} \tag{9}$$

and

$$c_{t+1}^r = R_{t+1}(1-\gamma)w_t^r. \tag{10}$$

We assume that $[p(f_t) + (1 - p(f_t)\mu)]zn_t^u < 1$ holds. This ensures that the savings, s_t^u, in (3) is positive. $\bar{z}n_t^r < 1$ is also assumed for $s_t^r > 0$. These restrictions are attributed to the fundamental assumption that one unit of time endowed to each individual cannot be lent or borrowed between individuals. The regional fertility gap seen in (5) and (9) captures that in the reality illustrated in Figure 2.

As the adult population of the economy in period t is simply the total population of the generation born in period $t-1$, N_t, we define ϕ_t as the ratio of the adult population in region u to the adult population of the economy, that is, N_t^u/N_t. As the economy consists of two regions, two hypothetical population distributions between the regions should be considered. The first is expressed as $\phi_t = 1$, where all individuals reside in region u. We define this situation as "concentration". The other is $0 < \phi_t < 1$, where some individuals reside in region u and others reside in region r. This is denoted as "dispersion". In any period t, adults choose their residential regions to maximize their lifetime utility. By substituting (5) and (6) into (1), we obtain the indirect utility function of each individual in region u as follows.

$$V_t^u = \ln\left(\frac{\gamma}{[p_t+(1-p_t)\mu]z}\right)^{\gamma}(R_{t+1}(1-\gamma)w_t^u)^{1-\gamma}. \tag{11}$$

From (1), (9) and (10), the indirect utility function of each individual in region r can be expressed as follows.

$$V_t^r = \ln\left(\frac{\gamma}{\bar{z}}\right)^{\gamma}(R_{t+1}(1-\gamma)w_t^r)^{1-\gamma}. \tag{12}$$

Assuming that individuals can migrate between the regions without costs, the utility levels equalize between the regions *when dispersion occurs*. When $V_t^u = V_t^r$ holds, we obtain the following no-arbitrage condition for a dispersion economy. [8]

$$\frac{w_t^u}{w_t^r} = \left(\frac{\mu}{(1-\mu)p_t+\mu}\right)^{\frac{-\gamma}{1-\gamma}}. \tag{13}$$

As a result of utility maximization, we find two notable outcomes. One outcome is the regional fertility gap. Note that $\bar{z} = \mu z$ is assumed. [9] From (5) and (9), we have

$$n_t^u < n_t^r. \tag{14}$$

The economic intuition for this result is as follows. The expected unit childcare cost in region u is higher than that in region r. Thus, parents in region u choose lower fertility as the optimum.

The other outcome is the regional savings gap. By substituting (5) and (9) into (3) and (7), respectively, we obtain

$$s_t^u = (1-\gamma)w_t^u \tag{15}$$

and

$$s_t^r = (1-\gamma)w_t^r. \tag{16}$$

As (13) shows that $w_t^r < w_t^u$, it is assured that

$$s_t^u > s_t^r. \tag{17}$$

The intuition for the optimal savings result is as follows. $n_t^u < n_t^r$ shown in (14) requires $c_t^u > c_t^r$ when the utilities in regions u and r are equal. As the preferences and rates of return on capital are common across the regions, the savings in region u must be higher than those in region r for $c_t^u > c_t^r$.

---

[8] As will be discussed later, when the equality in (13) does not hold but $V_t^u > V_t^r$ does, all individuals live in region u.

[9] If the assumption concerning unit childcare cost is relaxed appropriately, any cases of the regional fertility gap are feasible, and more complicated analyses can be demonstrated with other assumptions; however, these do not reflect the reality of the regional fertility gap.

2.2 Production

Let us suppose that final goods are produced in regions u and r. Although the goods are homogeneous, the production technologies are assumed to be different between the regions such that production is capital-intensive in region u and labor-intensive in region r. To maintain the transparency of the analysis, we assume that production in region u requires capital and labor as input factors, whereas production in region r uses only labor as an input factor. Based on these assumptions, we specify the aggregate production function as follows.

$$Y_t = A[K_t^{\alpha}(L_t^u)^{1-\alpha} + bL_t^r],\ \alpha \in (0,1),\ A, b > 0 \tag{18}$$

where K_t denotes the capital stock of the economy and L_t^i represents the labor supplied by individuals in region $i(= u, r)$. Assuming perfectly competitive markets for goods and factors, the returns on labor and capital are given by

$$w_t^u = (1-\alpha)(1-\tau)AK_t^{\alpha}(L_t^u)^{-\alpha}, \tag{19}$$

$$w_t^r = Ab, \tag{20}$$

$$R_t = \alpha(1-\tau)AK_t^{\alpha-1}(L_t^u)^{1-\alpha}, \tag{21}$$

where τ represents the tax rate charged by the regional government on output in region u. From (19) and (20), the relative labor demand function is expressed as follows.

$$\frac{w_t^u}{w_t^r} = b^{-1}(1-\tau)(1-\alpha)K_t^{\alpha}(L_t^u)^{-\alpha}. \tag{22}$$

2.3 Government

We assume that the regional government that provides childcare facilities in region u uses all the revenue from taxation on production in region u. Letting G_t represent government spending in region u in period t, the balanced budget constraint of the regional government in region u in period t is given by

$$G_t = \tau AK_t^{\alpha}(L_t^u)^{1-\alpha}.$$

The regional government provides childcare facilities to reduce the shortage of childcare services in region u. We consider the supply of childcare facilities effective if it reduces the probability that each parent has waitlisted children. We denote the effective supply of childcare facilities as f_t. With g_t representing government spending per adult in region u in period t, we assume that f_t increases in proportion to the increase in g_t; that is,

$$f_t = \lambda g_t,$$

where $g_t = G_t/N_t^u$. This is the production function for effective childcare facilities, where $\lambda > 0$ reflects the efficiency of government provision. [10] As p_t is positive and less than unity, we specify it as $p_t = {f_t}^{-1}$. [11] By substituting the optimal labor supply in region u, $(1-\gamma)\phi_t N_t$, into L_t^u, p_t is obtained as follows. [12]

$$p_t = (\lambda A)^{-1}(1-\gamma)^{\alpha-1}\tau^{-1}\left(\frac{k_t}{\phi_t}\right)^{-\alpha}, \tag{23}$$

where k_t denotes per-adult capital stock, defined as K_t/N_t. Although social welfare or economic growth in the long run are the most important policy goals, faced with the reality of urgent problems including low fertility or population decline, we observed that governments attempt to increase fertility with various supporting measures for having and rearing children. In this sense, the real government is a somewhat myopic decision-maker. We assume that the government's objective in this model is not to increase regional or social welfare but to increase regional fertility. Moreover, as a myopic decision-maker, the regional government is assumed to set the tax rate by taking K_t, N_t, and ϕ_t as given when it administrates the childcare policy. Hence, this myopic regional government believes that increasing the tax rate increases the supply of childcare facilities, lowers p_t, and raises the fertility rate in region u, as shown by the budget constraint of the government and (23). [13]

---

[10] It is natural to believe that λ decreases as the diversity of geographical relationship between office and home (e.g., commuting direction and commuting distance) increases, implying greater difficulty in reducing the shortage of childcare facilities owing to, for example, a "spatial mismatch" in the supply and demand of childcare facilities. f_t is assumed to depreciate perfectly within a period.

[11] The assumption of $p_t < 1$ gives the lower bound for the equilibrium capital-labor ratio in region u: $(\lambda A(1-\gamma)^{1-\alpha}\tau)^{\frac{-1}{\alpha}} < \frac{k_t}{\phi_t}$.

[12] The formulation is not only simple but also practically valid because the standby probability can never be zero. In reality, there are mismatches regarding the location of childcare facilities and content of the services provided. Consequently, an increase in childcare facilities does not always alleviate the problem of waitlisted children. For an empirical study on location mismatches, see Kawabata(2014).

[13] This paper does not consider the optimal policy issue.

3. Equilibrium

This section discusses the market equilibrium, which describes the determination of ϕ_t, p_t, m_t, K_t, and N_t. We also examine the short-run effect of a tax increase on ϕ_t and p_t taking K_t and N_t as given. Although ϕ_t is determined by individuals' residential choices, it must also clear the labor markets in both regions. Given $\phi_t N_t$, individuals' optimal labor supply by utility maximization, $(1-\gamma)$, forms the labor supply in region u, $(1-\gamma)\phi_t N_t$, whereas labor demand in region u, L_t^u, is optimally determined by profit maximization. Substituting the labor market clearing condition $(1-\gamma)\phi_t N_t = L_t^u$ into (22), we obtain

$$\frac{w_t^u}{w_t^r} = b^{-1}(1-\tau)(1-\alpha)\left[\frac{k_t}{(1-\gamma)(1-\sigma)\phi_t}\right]^{\alpha}. \tag{24}$$

Using (13), (23), and (24), ϕ_t in equilibrium is shown to be a linear function of k_t as follows.

$$\phi_t = \Omega(\tau)k_t, \tag{25}$$

where $\Omega(\tau)$ is characterized by the parameters in (13) and (24). [14] According to (25), ϕ_t increases in proportion to k_t; however, the upper bound of ϕ_t is unity by definition. (25) shows that when k_t increases, w_t^u increases in the labor market, more individuals live in region u, and a threshold value exists for w_t^u such that all individuals live in region u. Let $\bar{k}$ represent the value of k_t where the value of ϕ_t in (25) is unity. That is, $\bar{k}$ is defined as

$$\bar{k} = \Omega(\tau)^{-1}. \tag{26}$$

If k_t exceeds $\bar{k}$, ϕ_t is set to unity, and all individuals reside in region u. Otherwise, ϕ_t is related to k_t as in (25). Noting that $\bar{k}$ is a function of τ, that is, $\bar{k}(\tau)$, the determination of ϕ_t can be summarized as follows.

$$\phi_t = \begin{cases} \Omega(\tau)k_t & \text{if} \, k_t < \bar{k}(\tau), \\ 1 & \text{if} \, k_t \geq \bar{k}(\tau). \end{cases} \tag{27}$$

[14] Ω depends not only on τ but also on the other parameters included in (50) in the Appendix. However, we denote this simply as $\Omega(\tau)$ because we focus on the policy change captured by a change in τ. See the Appendix for the derivation of (25).

Thus, ϕ_t and p_t are uniquely determined in equilibrium with a given k_t, respectively.[15] An economy with $k_t < \bar{k}(\tau)$ exhibits dispersion, whereas an economy with $k_t \geq \bar{k}(\tau)$ displays concentration. Notably, the capital-labor ratio in region u, $\frac{K_t}{L_t^u}$, is always constant at $\bar{k}(\tau)$ when the economy exhibits dispersion, that is, when $0 < \phi_t < 1$, whereas it is k_t when the economy displays concentration, that is, when $\phi_t = 1$. Therefore, p_t can be easily proven to take a constant value independent of k_t in the dispersion interval $k_t < \bar{k}(\tau)$. It decreases as k_t increases in the concentration interval $k_t \geq \bar{k}(\tau)$. In addition to these properties, the fertility and wages in region u take constant values in the dispersion interval $k_t < \bar{k}(\tau)$ and increase as k_t increases in the concentration interval $k_t \geq \bar{k}(\tau)$. The determination of ϕ_t in the process of economic development can explain the reigonal gap in labor force illustelated in Figure 3.

As ϕ_t is determined such that the no-arbitrage condition on residential choice and the labor market equilibrium condition, both of which depend on τ, are satisfied, a change in τ should affect ϕ_t through multiple channels.

Considering (13), (23), and (24), we obtain the derivative of ϕ_t with respect to τ from (25) and assume its sign to be positive.

$$\frac{\partial \phi_t}{\partial \tau}\Big|_{k_t \ is \ given} = \frac{\frac{1}{1-\tau} - \frac{\gamma}{\tau} \frac{(1-\mu)p_t}{(1-\gamma)\mu + (1-\mu)p_t}}{\frac{\alpha}{\phi_t}\left[1 + \gamma \frac{(1-\mu)p_t}{(1-\gamma)\mu + (1-\mu)p_t}\right]} > 0. \quad (28)$$

The sign of the denominator is positive, but that of the numerator is indeterminate. Thus, the sign of (28) is generally indeterminate: given k_t, an increase in τ may increase or decrease ϕ_t. The economic intuition is as follows. The tax increase lowers both the expected unit childcare cost in region u and the after-tax marginal product of labor in region u. The former enhances the attractiveness of region u as a place to raise children, incentivizing individuals to live in region u. The latter diminishes the demand for labor, making it more difficult for individuals to work in region u. If the former outweighs the latter, a tax increase enhances the concentration ratio. This is a desirable situation in which the policy leads to a population increase in region u with N_t given.[16]

With $\frac{\partial \phi_t}{\partial \tau} > 0$, (25) and (26) show that

[15] Note that $p_t < 1$ gives the lower limit of $\bar{k}(\tau)$ as $(\lambda A((1-\sigma)(1-\gamma))^{1-\alpha}\tau)^{\frac{-1}{\alpha}}$.

[16] This situation can be naturally denoted as a desirable or favourable one for a regional government facing a severe population decline.

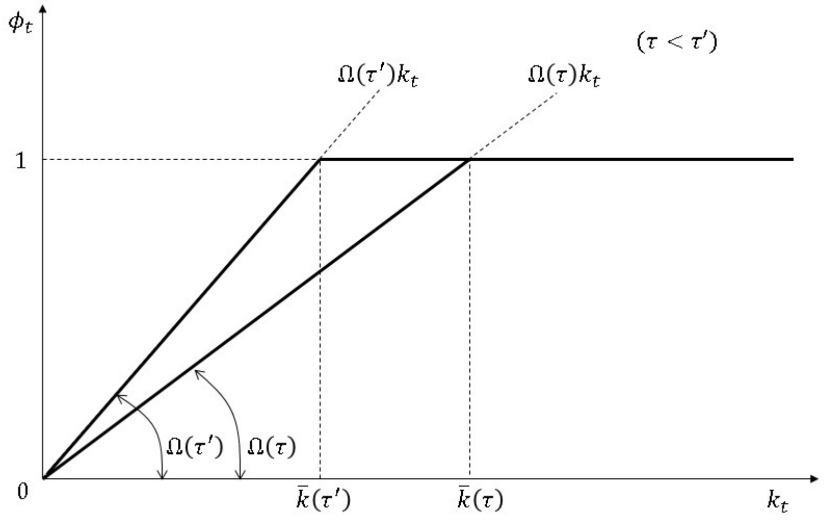
$$\frac{\partial\Omega(\tau)}{\partial\tau} > 0 \tag{29}$$

and

$$\frac{\partial\bar{k}(\tau)}{\partial\tau} < 0, \tag{30}$$

respectively. Figure 4 illustrates the relations among (28), (29), and (30).

Figure 4. Impact of τ on ϕ_t



Recalling that ϕ_t is a function of τ with k_t as given, the differentiation of (23) with respect to τ yields

$$\frac{\partial p_t}{\partial\tau} = p_t\left(\frac{\alpha}{\tau}\right)\left[\frac{\frac{\partial\phi_t}{\phi_t}}{\frac{\partial\tau}{\tau}} - \frac{1}{\alpha}\right]. \tag{31}$$

If $\frac{\partial\phi_t}{\partial\tau} > 0$ holds, as assumed thus far, then the sign of (31) is indefinite. In the following analysis, we assume that $\frac{\frac{\partial\phi_t}{\phi_t}}{\frac{\partial\tau}{\tau}} < \frac{1}{\alpha}$ such that

$$\frac{\partial p_t}{\partial\tau} < 0 \tag{32}$$

holds. Noting that $\phi_t = 1$ in concentration, the corresponding part for the case of $k_t \geq \bar{k}(\tau)$ is

$$\frac{\partial p_t}{\partial\tau} < 0. \tag{33}$$

The case of $\frac{\partial p_t}{\partial \tau} < 0$ indicates that a tax increase policy raises the fertility in region u, resulting in regional population growth.

In the subsequent analysis, we assumed that the tax increase policy increases ϕ_t and lowers p_t; the regional government can implement the policy as desired in the short run. [17] The discussion above is summarized as follows. With a given k_t, the myopic regional government can set τ such that both ϕ_t and n_t^u increase. This concerns the short-run effect of the policy in the sense that k_t is given. We stress that our analysis is conducted only in the case where the childcare support policy in region u by the myopic regional government of region u (i.e., the policy that aims at both an increase in n_t^u and in ϕ_t) is successful in the short run, and we examine whether regional and/or social welfare in the long run also increase in that case. [18] As the dynamic side of the model implies, k_t generally changes with population growth and capital accumulation in the long run; Figure 4 shows that when k_t falls, in the long run ϕ_t reaches less than the level before the policy is implemented, and, when that is the case, (23) shows a possible decline in the fertility rate in region u.

We express the total adult population in period $t+1$, N_{t+1}, as

$$N_{t+1} = [n_t^u \phi_t + n_t^r (1-\phi_t)] N_t. \tag{34}$$

Denoting the total fertility of the economy in period t, $\frac{N_{t+1}}{N_t}$, as m_t, (34) is revised to

$$m_t = \phi_t (n_t^u - n_t^r) + n_t^r. \tag{35}$$

Given τ, the dynamics of m_t is non-monotonic in k_t. As shown in the previous sections, ϕ_t increases proportionally to k_t, and n_t^u and n_t^r are constant in dispersion; if ϕ_t is unity, n_t^u increases in concentration as k_t increases because p_t decreases as k_t increases in that case. Considering these properties, together with the negative sign of $(n_t^u - n_t^r)$, it is straightforward to show that m_t decreases proportionally to k_t in dispersion, whereas $m_t (= n_t^u)$ increases as k_t increases in concentration. Figure 5 illustrates the dynamics of m_t.

[17] Hashimoto and Naito (2023) focused on this case, but they did not conduct a welfare analysis. This study discusses its impact on social welfare in detail and attempts to sound a warning about the nature of policies that focus only on population and fertility rates, regardless of how serious the population decline will be.

[18] We demonstrate that the policy leads to situations other than those in this case. It is sufficient to deal only with this case to show that the potential differences between the short- and long-run, or between population growth and welfare improvements in regional policies and their desired effects for the region, may not be so for the economy as a whole.

Figure 5. Total fertility m_t

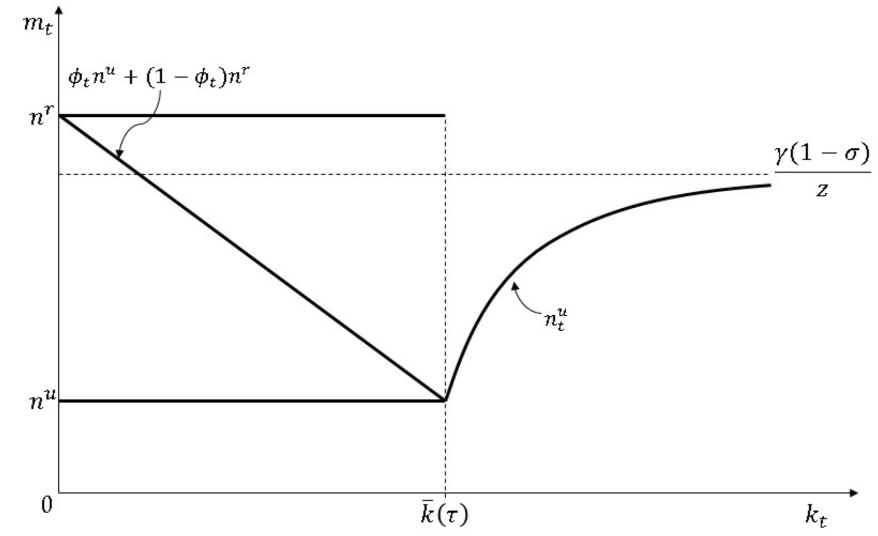
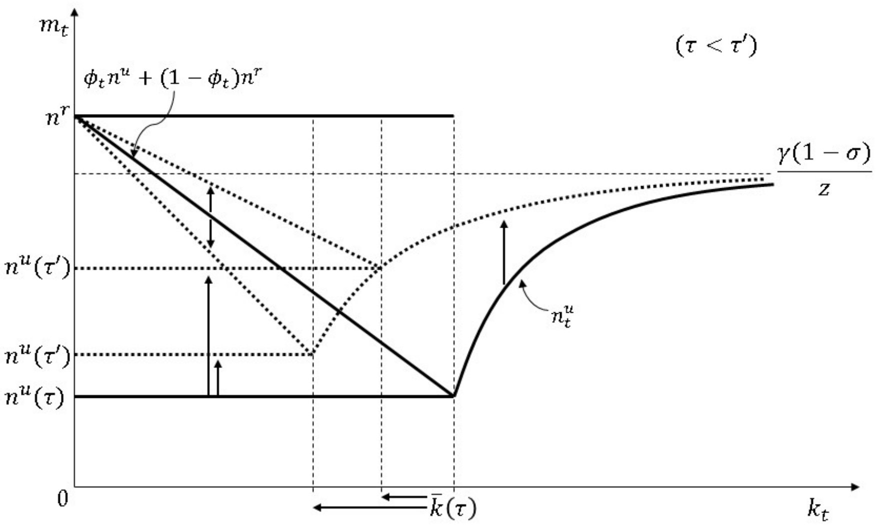


Figure 6. Impact of τ on m_t



Given k_t, although m_t changes with τ, the relationship is complex. Denoting ϕ_t and n_t^u as $\phi_t(\tau)$ and $n^u(\tau)$, respectively, the differentiation of (35) with respect to τ yields

$$\frac{\partial m_t}{\partial \tau} = \phi_t(\tau)\frac{\partial n^u(\tau)}{\partial \tau} + \frac{\partial \phi_t(\tau)}{\partial \tau}(n^u(\tau) - n^r).$$

As shown, the total effect of an increase in τ can be interpreted as a combination of the following two effects. One is the effect on an individual's fertility rate, which the first term in the right-hand side of the above derivative represents, and the other is the effect on interregional population migration, which the second term in the right-hand side represents. Hereafter, we refer to the former as "the individual effect" and the latter as "the migration effect." The individual effect is positive in that m_t increases as τ increases, whereas the migration effect is negative in that m_t decreases as τ increases. It is worthwhile to provide an intuitive explanation for the migration effect. $(n^u(\tau) - n^r)$ always holds, as shown in (14); thus, an increase in the ratio of the adult population in region u to that in region r increases the number of parents with fewer children and decreases the number of parents with more children. In concentration, an increase in τ negatively impacts the standby probability p_t as in (33), and thus increases total fertility m_t $(= n_t^u)$ as the migration effect does not occur by the definition of concentration. However, in dispersion, an increase in τ shapes the variations in outcomes with respect to changes in m_t. Moreover, the key factor shaping this contrast is interregional migration. This suggests that interregional migration is an important factor for a better understanding of demographics.

3.1 Capital accumulation

The goods market in period t clears when the aggregate savings in period t are equal to the aggregate investment in period t. Recalling that capital depreciates perfectly in a given period, we obtain the following goods market clearing condition in period t:

$$K_{t+1} = s_t^u N_t^u + s_t^r N_t^r. \tag{36}$$

Dividing both sides of (36) by N_{t+1} and using (35), (36) is revised as follows.

$$k_{t+1} = \frac{\phi_t s_t^u + (1-\phi_t) s_t^r}{m_t}. \tag{37}$$

Considering that ϕ_t, s_t^u, s_t^r, and m_t depend on k_t and τ, we obtain k_{t+1} as a function of k_t and τ, which is a nonlinear difference equation. We express it as

$$k_{t+1} = \Psi(k_t; \tau). \tag{38}$$

Two shapes of Ψ depend on k_t because ϕ_t depends on k_t from (27): the case of $k_t \geq \bar{k}$ and that of $k_t < \bar{k}$. Thus, each of these two cases must be considered separately. For $k_t < \bar{k}$, the population distribution displays dispersion, and for $k_t \geq \bar{k}$, it exhibits concentration.

Dispersion case: $\boldsymbol{k_t < \bar{k}}$

We prove the convexity of $\Psi(k_t; \tau)$ for $k_t < \bar{k}$. When k_t is smaller than $\bar{k}$, ϕ_t is not unity and is defined as $\Omega(\tau) k_t$ given by (27). Moreover, the equilibrium values for p_t, w_t^u, and thus s_t^u are constant in this case because k_t/ϕ_t in (23) and (19) is constant at $\bar{k}(\tau)$ or $\Omega(\tau)^{-1}$. Noting that p_t and s_t^u are independent of k_t, the differentiation of (37) with respect to k_t is as follows.

$$\frac{dk_{t+1}}{dk_t} = \frac{1}{m_t^2}\left\{\frac{\partial \phi_t}{\partial k_t}(s_t^u - s_t^r)m_t - [\phi_t s_t^u + (1-\phi_t)s_t^r]\frac{\partial m_t}{\partial k_t}\right\} > 0. \tag{39}$$

As $(s_t^u - s_t^r)$ is positive, $\frac{\partial s_t^u}{\partial k_t}$ is zero, and $\frac{\partial m_t}{\partial k_t}$ is negative, the sign of (39) is strictly positive. Moreover, the differentiation of (39) with respect to k_t is

$$\frac{d^2 k_{t+1}}{dk_t^2} = -2\frac{1}{m_t^3}\frac{\partial m_t}{\partial k_t}\left\{\frac{\partial \phi_t}{\partial k_t}(s_t^u - s_t^r)m_t - [\phi_t s_t^u + (1-\phi_t)s_t^r]\frac{\partial m_t}{\partial k_t}\right\} > 0. \tag{40}$$

We now define (38) for $k_t < \bar{k}$ as $\Psi_1(k_t;\tau)$, which is an increasing and convex function of k_t.

Concentration case: $k_t \geq \bar{k}$

We confirm the concavity of $\Psi(k_t;\tau)$ when $k_t \geq \bar{k}$. As given by (27), ϕ_t equals unity for $k_t \geq \bar{k}$. Substituting (15) and (16) into (37), (37) is revised to

$$k_{t+1} = \frac{Az}{\tau\gamma}(1-\alpha)(1-\tau)(1-\gamma)^{1-\alpha}(B(1-\mu) + k^\alpha \tau\mu), \tag{41}$$

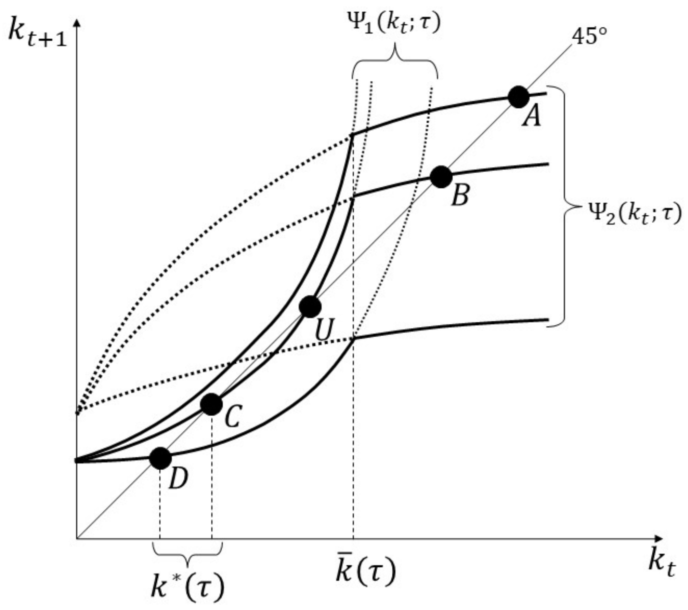
where $B \equiv (\lambda A)^{-1}(1-\gamma)^{\alpha-1}$. We define (38) for $k_t \geq k$ as $\Psi_2(k_t;\tau)$. Assumption $\alpha \in (0,1)$ in (18) proves that $\Psi_2(k_t;\tau)$ is an increasing and concave function of k_t. As previously mentioned, (38) can be summarized as follows.

$$\Psi(k_t;\tau) = \begin{cases} \Psi_1(k_t;\tau) & \text{if} k_t < \bar{k}(\tau) \\ \Psi_2(k_t;\tau) & \text{if} k_t \geq \bar{k}(\tau), \end{cases} \tag{42}$$

where $\Psi_1(0;\tau) = \frac{Ab(1-\gamma)\bar{z}}{\gamma} > 0$ and $\Psi_2(0;\tau) = \frac{z(1-\mu)(1-\alpha)(1-\tau)(\lambda\tau)^{-1}}{\gamma} > 0.$ [19]

[19] It is useful to know $\Psi_j(0;\tau)(j = 1,2) > 0$ to illustrate the graph of $\Psi_j(k_t;\tau)$ in the subsequent analysis.

Figure 7. Possible cases of steady states



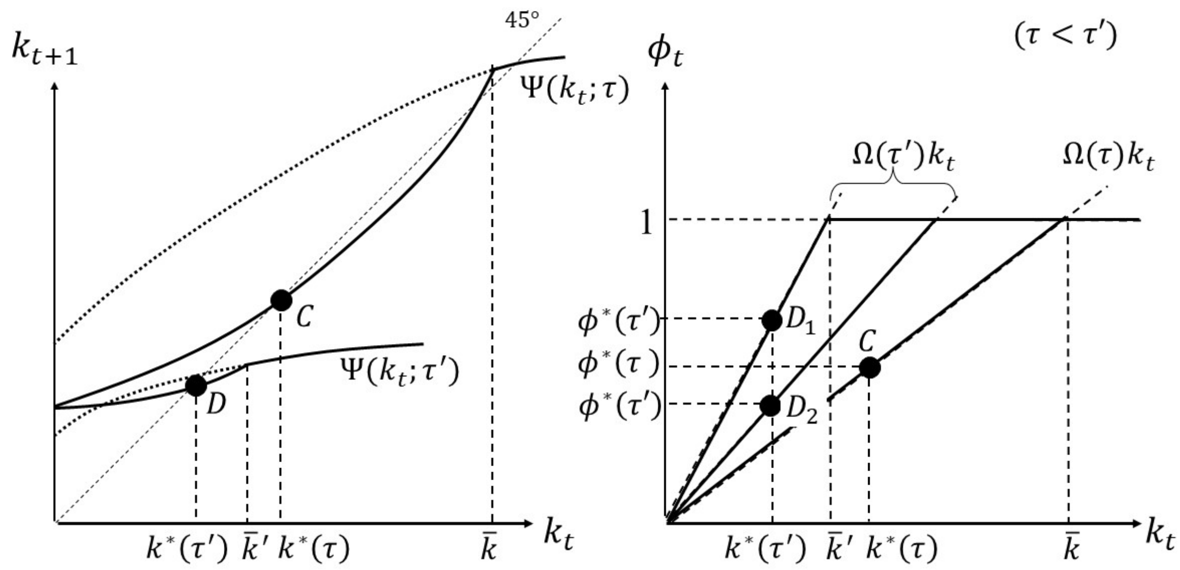
3.2 Steady state

We define the steady state as an equilibrium path where capital per adult k_t takes a constant value over time. Let $k(\tau)$ represent capital per adult when the tax rate is τ. Moreover, we denote $k(\tau)$ at the dispersion steady state as $k^*(\tau)$ or k^*. Once $k(\tau)$ is determined, the steady state values of the other endogenous variables are determined. Given τ, the steady state values of ϕ_t, p_t, n_t^u, and m_t are $\phi^*(\tau)$, $p^*(\tau)$, $n^{u*}(\tau)$, and $m^*(\tau)$, respectively. As $\Psi(k_t;\tau)$ is an increasing S-shaped function with a fixed positive intercept on the vertical axis and $\lim_{k_t \to +\infty} \frac{d^2 k_{t+1}}{dk_t^2} = 0$, the dynamic system will either depict an economy with one stable steady state, or that with two stable steady states and one unstable steady state, depending on parameters such as τ. The results are shown in Figure 7. There are two cases for economies with a unique steady state. One is an economy with a unique steady state characterized by concentration (Point A in Figure 7), and the other is an economy that has a unique steady state characterized by dispersion (Point D in Figure 7). These steady states are globally stable; thus, for an economy with any initial value of k_t, k_0 converges to a unique steady state; the stability of Point D is assured by assuming $\frac{dk_{t+1}}{dk_t}$<1 at $k_t = k^*(\tau)$ (Point D in Figure 7). In case of multiple steady states, one of the two stable steady states is concentration (Point B in Figure 7), and the other is dispersion (Point C in Figure 7); the unstable steady state (Point U in Figure 7) is dispersion. Although an equilibrium path has various patterns depending on k_0 and parameters such as τ, it is uniquely determined when k_0 and τ are given. Any steady state is considered with an appropriate k_0.

4. Policy effects on the steady state value: $\phi^*(\tau)$, $p^*(\tau)$, $n^{u*}(\tau)$, and $m^*(\tau)$

This section considers the impact of promoting childcare policy on $\phi^*(\tau)$, $p^*(\tau)$, $n^{u*}(\tau)$, and $m^*(\tau)$, which are the values at the dispersion steady states. As the Appendix shows, the sign of $\frac{\partial \Psi_1(k_t;\tau)}{\partial \tau}$ is indefinite, whereas that of $\frac{\partial \Psi_2(k_t;\tau)}{\partial \tau}$ is definitely negative. Since we focus only on the case where $\Psi(k_t;\tau)$ is a continuous function, we assume that $\frac{\partial \Psi_1(k_t;\tau)}{\partial \tau} < 0$ holds. As the promotion of childcare policy rotates $\Psi_1(k_t;\tau)$ clockwise around the vertical intercept and shifts $\Psi_2(k_t;\tau)$ downward, maintaining the continuity of $\Psi(k_t;\tau)$, the dispersion steady state is supposed to move to a new dispersion steady state.

Figure 8. Possible cases of steady states



Impact of τ on $\phi^*(\tau)$

The impact of τ on $\phi(\tau)$ is not uniquely determined when the steady state at Point C moves to that at Point D. As $\bar{k}$ decreases with an increase in τ, the steady state at Point C may move to either D_1 or D_2 in Figure 8. This is because τ has two types of impacts, as in (27). One is the impact on Ω, and the other is the impact on k^*. The former has a positive effect on Ω, whereas the latter has a negative effect on k^*; the impact of τ on $\phi(\tau)$ is ambiguous. An increase in τ does not necessarily increase the ratio of the adult population in region u in the long run.

Impact of τ on $p^*(\tau)$

We now consider the impact on $p^*(\tau)$. Using (29), (23) shows that the sign of $\frac{\partial p^*(\tau)}{\partial \tau}$ is negative, whereas that of $\frac{\partial \phi(\tau)}{\partial \tau}$ is ambiguous. Childcare policy reduces the standby probability not only in the short run but also in the long run.

Impact of τ on $n^{u*}(\tau)$

Considering $\frac{\partial p^*(\tau)}{\partial \tau} < 0$, (5) shows that $\frac{\partial n^{u*}}{\partial \tau} > 0$ holds. Childcare policy reduces fertility in region u not only in the short run but also in the long run.

Impact of τ on $m^*(\tau)$

Finally, we consider the impact on $m^*(\tau)$. From (35), the total fertility at the steady state, $m^*(\tau)$ is given as $m^*(\tau) = \phi * (\tau)(n^{u*}(\tau) - n^r) + n^r$. By differentiating $m^*(\tau)$ with respect to τ, we obtain:

$$\frac{\partial m^*(\tau)}{\partial \tau} = \frac{\partial \phi^*(\tau)}{\partial \tau}(n^{u*}(\tau) - n^r) + \frac{\partial n^{u*}(\tau)}{\partial \tau}. \tag{43}$$

The sign of $\frac{\partial m^*(\tau)}{\partial \tau}$ is indefinite despite the fact that $\frac{\partial n^{u*}(\tau)}{\partial \tau} > 0$. The ambiguity of the sign of $\frac{\partial \phi^*(\tau)}{\partial \tau}$ leads to the result. The promotion of childcare policy does not always improve the total fertility rate.

One significant result of the analysis conducted is that the regional childcare policy in region u can increase fertility in region u in both the short and long terms. The policy is successful for regional governments whose objective is to increase regional fertility.

5. Welfare

5.1 Impact of τ on regional and social welfare

This section considers the impact of childcare policy on increasing τ by comparing the regional and social welfare at the steady states. First, we consider the welfare of each region. We define the indirect utility of individuals in each region as regional welfare. As the indirect utilities in both regions are equal under the dispersion equilibrium ($V^u = V^r$), V^r is given by

$$V^r(\tau) = \gamma \ln n^{r*}(\tau) + (1-\gamma)\ln c^{r*}(\tau), \tag{44}$$

where $n^{r*}(\tau)$ and $c^{r*}(\tau)$ denote the number of children and retirement consumption in region r at a steady state, respectively. From (9) and (10), (44) is revised as

$$V^r(\tau) = \gamma \ln\left(\frac{\gamma}{\bar{z}}\right) + (1-\gamma)\ln[R^*(1-\gamma)Ab]. \tag{45}$$

By differentiating (45) with respect to τ, the impact of τ on $V^r(\tau)$ is given by

$$\frac{\partial V^r(\tau)}{\partial \tau} = \frac{1-\gamma}{R^*}\frac{\partial R^*}{\partial \tau} \gtreqless 0. \tag{46}$$

Using (19) and (23), we write R^* as

$$R^* = A\alpha(1-\tau)(1-\gamma)^{1-\alpha}\left(\frac{1}{\Omega(\tau)}\right)^{\alpha-1}. \tag{47}$$

By differentiating (47) with respect to τ, we obtain

$$\frac{\partial R^*}{\partial \tau} = A\alpha(1-\gamma)\left\{-\left(\frac{1}{\Omega(\tau)}\right)^{1-\alpha} - (1-\tau)(\alpha-1)(\Omega(\tau))^{\alpha}\frac{\partial \Omega(\tau)}{\partial \tau}\right\} \gtreqless 0. \tag{48}$$

Assuming $\frac{\partial \Omega(\tau)}{\partial \tau} > 0$, the sign of $\frac{\partial R^*}{\partial \tau}$ is indeterminate. Consequently, the sign of $\frac{\partial V^r(\tau)}{\partial \tau}$ is indeterminate. Next, we define social welfare as W, which is the weighted average of each regional welfare $V^i(i = u, r)$ with ϕ as the weight; that is,

$$W \equiv \phi V^u + (1-\phi)V^r. \tag{49}$$

As V^u is equal to V^r, W in (49) can be reduced to V^r. Therefore, the results for regional welfare apply directly to the study of the impact of τ on W.

5.2 Discussion

The promotion of childcare policy in region u is designed to increase both the fertility rate of region u and the ratio of the population in region u to that in the economy in the short run, increase the fertility rate of region u in the long run, and increase the population ratio of region u in the long run. The indefinite impact of childcare improvement policy on social welfare is based on ϕ_t, which plays a key role in this study. This result is similar to that obtained for the total fertility rate.[20] Although our analysis is limited to the case in which the regional fertility policy succeeds, such a policy affects the path of capital accumulation, changes the capital intensity of production in region u because of the ratio of the adult population in region u, and alters its marginal products. Therefore, policy effects have various outcomes. The degree of change in the net interest rate, resulting from an increase in

[20] See Hashimoto and Naito (2023) for a detailed discussion of this mechanism.

marginal products of capital, is noteworthy. The promotion of childcare support policies in region u, as described above, has been shown to reduce both the regional and social welfare in region u. This implies a decrease in consumption that leads to a welfare loss, outweighing the welfare gain from having children. Although we do not refer to the case where the impact of policy decreases ϕ_t or does not affect it, an increase in τ in those cases always decreases marginal products and welfare because of declining net interest. Summarizing the above, a loss in social welfare comes from a significant decrease in consumption in retirement. Thus, the regional government needs to introduce other policies that increase the marginal productivity of net capital or reduce the decline in consumption in retirement in order for the regional social welfare to increase when the regional childcare support policies are implemented to raise the regional fertility. Theoretically, there is a steady state in which all individuals reside in region u. Although the social welfare function in this case is defined by the indirect utility function in region u, we can develop a similar argument.

As fot determination of τ, we have not analyzed the optimality for τ. First, the government determines τ without considering the reaction of individuals and firms to the governent policy. Therefore, we do not refer to the optimal rate of τ to maximize social welfare because this model is not set up properly to pursue the optimality for τ. If we refer to the optimal rate of τ, an additional assumption on the government behavior must be imposed. As for social welfare, we have another controversial issue to discuss. The issue is the difficulty in defining the social welfare in overlapping generations models. While we have defined social welfare based on the lifetime utility of any generation in the steady state, there are other definitions of social welfare. For instance, a weighted sum of the utilities of different generations in a given period in the steady state is considerable; it is not easy to determine the weight. It is interesting to discuss the determination of the weight because it is related to the silver democracy issue; the weight can be related to the ratio of the young to the elderly generation. It is also possible to define social welfare as the sum of the indirect utilities of individuals in both regions.

6. Conclusion

This study investigated the effect of regional childcare policy on welfare at steady state in an overlapping generations economy that consists of regions u and r, which complements Hashimoto and Naito (2023) who analyze its effect only on the fertility rate and capital accumulation. Reconfirming the existence of two types of steady state equilibria, we conducted a welfare analysis only in the case of the dispersion equilibrium, in which the two regions exist. We explored the effect of promoting childcare support in region u on welfare in the long run, limited to the case in which the policy works well as anti-population-decline in the sense that it raises both the fertility rate of region u and the population ratio of region u. The analysis clarified that the success of regional childcare support policy as an anti-population-decline policy did not guarantee an increase in welfare. Extending the analysis to other possible cases easily brings about various welfare effect outcomes. A primary factor shaping these outcomes is the variety of policy effects on population ratios in the short run. For regional population growth policies to be implemented in a way that results in welfare improvement, it is suggested that regional governments consider not only the improvement of childcare environment and boosting of fertility rates in their own regions but also the policy impact on the population distribution of the whole economy. This implies that the central government should implement policies that can target population distribution throughout the economy in coordination with population policies implemented by regional governments. Future studies should consider effective population growth policies based on intergovernmental cooperation and competition.

Acknowledgements

The previous version of this paper was presented at the 35th Annual Meeting of Applied Regional Science Conference in 2021 (online) and Keio University Applied Economics Workshop in 2021. We are grateful to Takaaki Takahashi, Masao Ogaki, Miki Seko, Kimiko Terai, Kozo Kiyota, and Mizuki Kawabata for helpful comments and discussions in these meetings. This work was supported by JSPS KAKENHI Grant Numbers 20K01693, 23K01354 and 24K04849.

References

Diaz Gandasegui, V., Elizalde-San Miguel, B. and Sanz, M.T., 2021. Back to the future: a sensitivity analysis to predict future fertility rates considering the influence of family policies? The cases of Spain and Norway. *Social Indicators Research*, 154(3), pp. 943–968. https://doi.org/10.1007/s11205-020-02566-7

Goto, H., and Minamimura, K. (2019). Geography and demography: New economic geography with endogenous fertility. *The Japanese Economic Review*. 70(4), pp.537-568. https://doi.org/10.1111/jere.12213

Hashimoto, H. and Naito, T., 2021. Urbanization, waitlisted children, and childcare support in a two-region overlapping generations model. *Regional Science Policy and Practice*, 13(3), pp.1068-1089. https://doi.org/10.1111/rsp3.12377

Hashimoto, H. and Naito, T., 2023. Does an urban childcare policy improve the nurturing environment in a city?. *Asia-Pacific Journal of Regional Science*, 8, pp.1-25. https://doi.org/10.1007/s41685-023-00316-4

Hashimoto, H. and Naito, T., 2024. Regional differences in childcare environment, urbanization, and fertility decline. *Letters in Spatial and Resource Sciences*, 17(1), 8. https://doi.org/10.1007/s12076-024-00369-9

Kawabata, M., 2014. Childcare access and employment: the case of women with preschool-aged children in Tokyo. *Review of Urban and Regional Development Studies*, 26(1), pp.40-56. https://doi.org/10.1111/rurd.12018

Kimura, M., and Yasui, D., 2007. Occupational choice, educational attainment, and fertility. *Economics Letters*, 94(2), pp. 228-234. https://doi.org/10.1016/j.econlet.2006.06.035

Kondo, K., 2018. Does agglomeration discourage fertility? Evidence from the Japanese general social survey 2000–2010. *Journal of Economic Geography*, 19(3), pp.677-704. https://doi.org/10.1093/jeg/lbx048

Koka, K. and Rapallini, C., 2023. Italy's demographic trap: voting for childcare subsidies and fertility outcomes. *European Journal of Political Economy*, 76, January. https://doi.org/10.1016/j.ejpoleco.2022.102264

Naito, T. and Omori, T., 2017. Aging and urban agglomeration under a multi-regional overlapping generations model. *Review of Urban and Regional Development Studies*, 29(2), pp. 135-150. https://doi.org/10.1111/rurd.12065

Sato, Y. and Yamamoto, K., 2005. Population concentration, urbanization, and demographic transition. *Journal of Urban Economics*, 58(1), pp. 45-61. https://doi.org/10.1016/j.jue.2005.01.004

Sato, Y., 2007. Economic geography, fertility and migration. *Journal of Urban Economics*, 61(2), pp.372-387. https://doi.org/10.1016/j.jue.2006.08.002

Schoppa, L.J., 2020. The policy response to declining fertility rates in Japan: relying on logic and hope over evidence. *Social Science Japan Journal*, 23(1), pp. 3–21. https://doi.org/10.1093/ssjj/jyz046

Yakita, S., 2011. Regional public goods, migration, and growth. *Letters in Spatial and Resource Sciences*, 4(2), pp.129-138. https://doi.org/10.1007/s12076-011-0057-9

Appendix

A. Derivation of (25)

Combining (13) with (22), we obtain

$$\frac{\phi_t}{k_t} = \left\{\frac{b}{(1-\tau)(1-\alpha)(1-\sigma)^{-\alpha}(1-\gamma)^{-\alpha}(1-\sigma)^{\frac{1}{1-\gamma}}}\left(\frac{1}{\mu}\right)^{\frac{\gamma}{1-\gamma}}[\mu+(1-\mu)p_t]^{\frac{\gamma}{1-\gamma}}\right\}^{\frac{-1}{\alpha}}. \tag{50}$$

Considering that p_t in (50) is a function of $\frac{\phi_t}{k_t}$ as in (23), the derivative of ϕ_t with respect to k_t is obtained as

$$\frac{d\phi_t}{dk_t} = \frac{\phi_t}{k_t}. \tag{51}$$

Using the implicit function theorem for the case of $\frac{\phi_t}{k_t} > 0$, ϕ_t can be expressed as a function of k_t:

$$\phi_t = \Phi(k_t;\tau).$$

Then, the integration of (51) shows that ϕ_t is a linear function of k_t such that

$$\phi_t = \Omega(\tau)k_t,$$

where $\Omega(\tau)$ is characterized by the parameters in (50).

B. Sign of $\frac{\partial\Psi}{\partial\tau}$

This section describes how an increase in τ shifts $\Psi(k_t;\tau)$.

B1. $k_t \geq \bar{k}$

Because ϕ_t is unity by definition, as in (27), for the case of $k_t \geq \bar{k}$, substituting $\phi_t = 1$ for (37), we obtain $k_{t+1} = \Psi_2(k_t;\tau)$ as follows.

$$k_{t+1} = \frac{s_t^u}{n_t^u}, \tag{52}$$

where s_t^u and n_t^u are defined as follows:

$$s_t^u = A(1-\alpha)(1-\tau)(1-\sigma)^{1-\alpha}(1-\gamma)^{1-\alpha}(k_t)^{-\alpha}$$

and

$$n_t^u = \left(\frac{\gamma(1-\sigma)}{z}\right)[p_t(\tau) + (1 - p_t(\tau))\mu]^{-1}.$$

By differentiating (52) with respect to τ given $k_t \geq \bar{k}$, we obtain

$$\frac{\partial k_{t+1}}{\partial \tau} = \frac{s_t^u}{n_t^u}\left[-(1-\tau)^{-1} + \left(\frac{1-\mu}{\mu + (1-\mu)p_t(\tau)}\right)\frac{\partial p_t(\tau)}{\partial \tau}\right]. \tag{53}$$

Since τ, μ, and p_t are positive and strictly less than unity, and $\frac{\partial p_t(\tau)}{\partial \tau}$ always has a negative sign in the case of $k_t \geq \bar{k}$ as shown in (33), the sign of (53) is definitely negative. Thus, an increase in τ always shifts down $\Psi_2(k_t; \tau)$.

B2. $k_t < \bar{k}$

Because the case of $k_t < \bar{k}$ is characterized by $0 < \phi_t < 1$, $k_{t+1} = \Psi_1(k_t; \tau)$ is given by (37); that is,

$$k_{t+1} = \frac{\phi_t s_t^u + (1-\phi_t)s_t^r}{\phi_t n_t^u + (1-\phi_t)n_t^r}, \tag{54}$$

where n_t^u is the same as for $k_t \geq \bar{k}$, but s_t^u differs as follows:

$$s_t^u = A(1-\alpha)(1-\tau)(1-\sigma)^{1-\alpha}(1-\gamma)^{1-\alpha}\left(\frac{k_t}{\phi_t}\right)^{-\alpha}.$$

Considering that s_t^r and n_t^r are independent of τ, the differentiation of (54) with respect to τ gives

$$\frac{\partial k_{t+1}}{\partial \tau} = \frac{1}{((n_t^u - n_t^r)\phi_t + n_t^r)^2}$$

$$\times\left\{\left[(s_t^u - s_t^r)\frac{\partial \phi_t}{\partial \tau} + \phi_t\frac{\partial s_t^u}{\partial \tau}\right]((n_t^u - n_t^r)\phi_t + n_t^r)\right.$$

$$\left.-((s_t^u - s_t^r)\phi_t + s_t^r)\left[(n_t^u - n_t^r)\frac{\partial \phi_t}{\partial \tau} + \phi_t\frac{\partial n_t^u}{\partial \tau}\right]\right\}, \tag{55}$$

where

$$\frac{\partial s_t^u}{\partial \tau} = \frac{-\alpha s_t^u}{\tau}\left[\frac{\tau}{\alpha(1-\tau)} + \frac{\tau}{\phi_t}\frac{\partial \phi_t}{\partial \tau}\right] \tag{56}$$

and

$$\frac{\partial n_t^u}{\partial \tau} = \left(\frac{-(1-\mu)}{\mu+(1-\mu)p_t(\tau)}\right) n_t^u \frac{\partial p_t(\tau)}{\partial \tau}. \tag{57}$$

From (31), we have

$$\frac{\partial \phi_t}{\partial \tau} = \frac{\phi_t}{\alpha\tau}\left(1 + \frac{\partial p_t}{\partial \tau}\frac{\tau}{p_t}\right). \tag{58}$$

Together with (56), (57), and (58), (55) can be revised as follows:

$$\frac{\partial k_{t+1}}{\partial \tau} = \frac{1}{((n_t^u - n_t^r)\phi_t + n_t^r)^2}\left\{(n_t^r s_t^u - n_t^u s_t^r)\frac{\phi_t}{\alpha\tau} + m_t\left(1 - \frac{\phi_t}{1-\tau}s_t^u\right)\right.$$

$$+\left[(n_t^r s_t^u - n_t^u s_t^r)\frac{\phi_t}{\alpha\tau} + m_t + ((s_t^u - s_t^r)\phi_t + s_t^r)\phi_t\right.$$

$$\left.\left.\times\left(\frac{1-\mu}{\mu+(1-\mu)p_t}\right)n_t^u\frac{p_t}{\tau}\right]\frac{\partial p_t}{\partial \tau}\frac{\tau}{p_t}\right\} \gtreqless 0. \tag{59}$$

As the sign of $\left(1 - \frac{\phi_t}{1-\tau}s_t^u\right)$ can be positive, negative, or zero, determining the sign of (59) is impossible. Thus, an increase in τ may shift $\Psi_1(k_t;\tau)$ up, down, or have no effect.